SPECIFYING NONLINEARITY AND HYSTERESIS OF ROTATING TORQUEMETERS

Nonlinearity and Hysteresis definitions were developed to characterize performance of common sensors such as scales and load cells. Those definitions have been widely used to characterize Torquemeters. Despite the similarities between Rotary Torquemeters and modern weighing devices, there are significant differences between them and their applications. Most important, **real-world Torque signals are invariably dynamic** whereas weighing, and most load measurements, are primarily static.

Thus, **when those definitions are applied to Torquemeters, the results can be misleading and, the conclusions based on them, wrong.** This note describes these definitions as well as others that more accurately characterize real-world performance of rotating Torquemeters, and provide the basis for more meaningful comparisons. It also discusses the impact Nonlinearity and Hysteresis have on accuracy.

NONLINEARITY

To avoid errors, a transducers output should faithfully reflect its input. When a transducer is nonlinear, its output contains errors in the magnitude of the average value as well as its dynamic components (inevitably present on drivelines; see Himmelstein Application Note 221101D). Nonlinearity also generates output signals not present on the driveline. Those include harmonics of the input, and sum and difference frequencies; see Appendix 1. This situation is exacerbated during transient conditions. The only **way to avoid such errors is to use a Torquemeter with minimal Nonlinearity.**

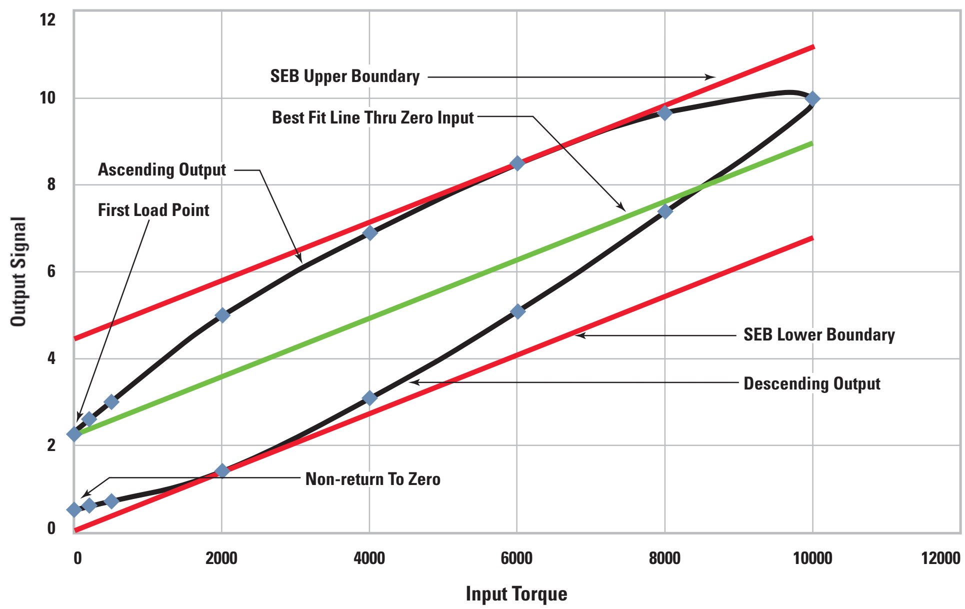
Using a computer or microprocessor-based signal conditioner, **it's possible to "linearize" the calibration stand response of a nonlinear transducer.** However, **this procedure is only valid when the torque signal is static,** i.e. has no sinusoidal components, perturbations, torque reversals, inertia torques, etc. – that's **never the case for a rotating measurement. Although "linearization" can yield a very linear response on a calibration stand, it will produce significant additional errors in real-world, rotating applications.**

Employing a "linearizer" for a rotating (dynamic) measurement generates additional errors and distortion components because it processes the distorted signal through another nonlinear element. Not only can it not remove the transducer's distortion components, but in fact, it increases their number. **Beware of a Rotary Torquemeter that includes linearization software. While it can improve the linearity of a static calibration, *it will generate additional errors in real-world, rotating applications.***

To determine a Torquemeter's Nonlinearity, one applies ascending and descending loads (usually 10 CW and 10 CCW) from a precise (low uncertainty), Accredited Torque Calibration Stand and records the outputs. Nonlinearity is the outputs' greatest deviation of ascending data from a reference line that best describes the Torquemeters' ascending response. It is expressed as a percentage of full scale.

The reference line often includes the end point but, it can intercept any one or two points. When it includes the end point, the result is called End Point Nonlinearity. End Point Nonlinearity is simple to evaluate even without a computer. However, **a line**

Torquemeter Input vs Output, Best Fit Line Thru Zero Input, and SEB Boundaries



through the end points is arbitrary and virtually never best characterizes the Torquemeter. As a result, when using the End Point, the sensors **reported Nonlinearity is invariably different than its true Nonlinearity.**

A more accurate result is obtained by using a Least Squares Line for the reference line. Although more difficult to compute, **the most accurate reference line is a Best Fit Line (BFL) based on both ascending and descending data.** For that reason, Himmelstein has elected to use a BFL for evaluating its Torquemeter products. **We define the BFL as that line which passes through the first load (zero input) point and has the smallest maximum deviation from the ascending and descending data points.** The diagram above is a typical calibration curve **exaggerated for clarity.** It includes ascending and descending data, a best fit reference line thru the first load point, and Static Error Band (SEB) boundaries (see following discussion).

As noted, the reference line could be End Point, Least Squares, Best Straight Line, etc. The output of any real world Torquemeter will deviate from a straight line. Conventionally, **the output's deviation from the straight line is determined for each *ascending* load point. Nonlinearity is defined as the greatest difference among all the *ascending* load points. The difference between the ascending and descending outputs at each load point is also calculated. A Torquemeters hysteresis is defined as the greatest difference between those values.** Nonlinearity and hysteresis are expressed as a percent of full scale. These values are separately determined for CW and CCW torque directions. A Torquemeters Nonlinearity and Hysteresis are quoted as the greater of the CW and CCW values.

A Torquemeter will never see an error equal to Hysteresis as defined above. The only Hysteresis error it sees is the deviation from the reference line, not the difference between ascending and descending values, Furthermore, in the conventional definition of Nonlinearity the descending response is ignored. Because rotating measurements are dynamic and include torque oscillations and reversals **the descending part of the response curve must be included to accurately depict Torquemeter performance.**

A practical solution is to calculate the **Static Error Band (SEB)**, as follows. Both ascending and descending data is used to compute a reference BFL through the first load point. Then the greatest deviation from that BFL is found. That determination is made for CW and CCW loadings. The Torquemeters' Combined Nonlinearity and Hysteresis, or Static Error Band, is the greater of the CW and CCW deviations. **This method has the advantage of accounting for ascending and descending response while intrinsically accounting for Hysteresis.** Himmelstein now uses this definition. Not only does it realistically account for both ascending **and** descending data, but the reference BFL is based on all the data points (usually 21) and closely matches the transducer in static and dynamic applications.

Calibration Data, 1,200,000 lbf-in Torquemeter (Calibration Serial Number 500288)

Input Torque (lbf-in)	Clockwise Output (mV/V)		Counterclockwise Output (mV/V)	
	Ascending	Descending	Ascending	Descending
0	-0.0004	0.0021	0.0002	0.0001
120,000	0.2729	0.2768	-0.2699	-0.2658
240,000	0.5428	0.5505	-0.5370	-0.5352
360,000	0.8151	0.8217	-0.8050	-0.8060
480,000	1.0876	1.0908	-1.0736	-1.0776
600,000	1.3600	1.3599	-1.3434	-1.3476
720,000	1.6318	1.6294	-1.6143	-1.6185
840,000	1.9018	1.8989	-1.8864	-1.8885
960,000	2.1711	2.1680	-2.1577	-2.1601
1,080,000	2.4393	2.4375	-2.4288	-2.4313
1,200,000	2.7080		-2.7012	

The table above summarizes the calibration of a non-Himmelstein Torquemeter performed in our laboratory. The next table illustrates results obtained by analyzing that calibration using reference lines based on End Point, BFL and Least Squares. Their use yields significant differences in calculated Nonlinearity and Combined Error. Based on the conventional definition (see above) the hysteresis of this device is: CW = 0.284%, CCW = 0.156%. When classifying modestly accurate Torquemeters such as this, valid comparisons can only be made when the most precise analysis method is used. In any case, you should know and understand the method used and its limitations.

Based on this data one could conclude this Torquemeter will have a measurement error of about 0.3%. But, that will only be true if the torque input is static. With dynamic signals present, the error will be greater and will include frequencies that aren't present on the driveline; see Appendix 1.

Finally, **unless the Torquemeter has adequate Overrange, the reported average torque near full scale will contain additional large errors and the distortion components will be greatly magnified.** All Himmelstein Digitally-based Torquemeters have between 150% and 300% Overrange, model dependent. Typical non-linearity in the Overrange region is 0.04%, maximum nonlinearity is 0.1%. See Himmelstein Application Note 20805B for more information on the critical importance of Torquemeter Overrange.

Analysis of 1,200,000 lbf-in Torquemeter (Calibration Serial Number 500288)

Reference Line →	End Point	Best Fit Thru First Load Point	Least Squares
CW Nonlinearity (% of Full Scale)	0.264*	0.165*	0.158*
CCW Nonlinearity (% of Full Scale)	0.263*	0.179*	0.141*
CW Combined Error (% of Full Scale)	0.354**	0.284***	0.237**
CCW Combined Error (% of Full Scale)	0.263**	0.179***	0.148**

* Based on ascending reference line and ascending data.
** Based on ascending reference line and ascending and descending data.
*** Reference Line and calculations are based on ascending and descending data.

Calibration data of a more accurate, Dual Range Bearingless Digital Torquemeter is shown in the following table. It is a Himmelstein Model MCRT 88707V(5-4)NN. The last table summarizes its performance based on different analysis methods. High precision devices are essentially identical when analyzed with each method. Based on the conventional definition (see above) the hysteresis of this device is: CW = 0.008%, CCW = 0.006%.

In the limit, if a Torquemeter were perfectly linear, then all valid analysis methods should produce precisely the same result, i.e., Nonlinearity, Hysteresis and Combined Error equal to zero. Nonetheless, the most accurate rotating Torquemeter results are based on the SEB method described above.

Calibration Data for 50,000 lbf-in Himmelstein Dual Range Bearingless Digital Torquemeter (Values are digital data scaled in engineering units, Data is for Torquemeters' high range) (Calibration Serial Number 3005844)

Clockwise			Counterclockwise		
Input Torque (lbf-in)	Torquemeter Output		InputTorque (lbf-in)	Torquemeter Output	
	Ascending	Descending		Ascending	Descending
0	1	1	0	-1	-1
4,997	4,994	4,997	4,997	-4,996	-4,996
9,994	9,991	9,993	9,994	-9,991	-9,991
14,991	14,986	14,990	14,991	-14,986	-14,988
19,988	19,982	19,986	19,988	-19,983	-19,984
24,984	24,978	24,981	24,985	-24,978	-24,981
29,981	29,974	29,978	29,982	-29,974	-29,977
34,978	34,969	34,972	34,979	-34,970	-34,973
39,975	39,965	39,969	39,976	-39,966	-39,968
44,972	44,962	44,964	44,973	-44,962	-44,963
49,969	49,958		49,970	-49,959	

Analysis of 50,000 lbf-in Himmelstein Dual Range Bearingless Digital Torquemeter (Data is for the high range) (Calibration Serial Number 3005844)

Reference Line ➔	End Point	Best Fit Thru First Load Point	Least Squares
CW Nonlinearity (% of Full Scale)	0.005*	0.005*	0.004*
CCW Nonlinearity (% of Full Scale)	0.005*	0.004*	0.003*
CW Combined Error (% of Full Scale)	0.006**	0.005***	0.005**
CCW Combined Error(% of Full Scale)	0.005**	0.004***	0.004**

* Based on ascending reference line and ascending data.
** Based on ascending reference line and ascending and descending data.
*** Reference Line and calculations are based on ascending and descending data

Appendix 1

How Nonlinearity Creates Amplitude Errors and Erroneous, Nonexistent Signals

A properly functioning Torquemeter's response curve is smooth, and without gaps. If that isn't the case, it is defective and should be repaired or replaced. Accordingly, a properly functioning Torquemeter's response can be described by the following power series.

$$\text{Output} = a_0 + a_1T_{in} + a_2T_{in}^2 + a_3T_{in}^3 + a_4T_{in}^4 + a_5T_{in}^5 + \quad (1)$$

Where a_0, a_1, a_2 etc. are constant coefficients and T_{in} is the input Torque.

If the Torquemeter is perfectly linear, then its response is described by the first two terms. The following table contains the coefficients for equations of the 1st through 5th order. It is based on least square calculations that include both the ascending and descending CW response of the 1,200,000 lbf-in Torquemeter previously discussed. Also included is the Akaike Info Criterion (AIC_c); the best model has the lowest AIC_c.

Based on the Akaike Info Criterion, it's clear the 2nd order equation is the best model for this Torquemeter. Therefore, for this analysis, we will use the 2nd order response. Further assume the dynamic input has an average value and two sinusoidal components. Thus, the input signal is:

$$T_{in} = T_a + T_b (\sin \omega_b t) + T_c (\sin \omega_c t) \quad (2)$$

Where T_a is the average value of the input torque and T_b is the peak amplitude of the sinusoidal component whose angular velocity is ω_b. Similarly, T_c is the peak amplitude of the sinusoidal component whose angular velocity is ω_c. If the Torquemeter were perfectly linear, it would output a scaled, mirror image of the input. That is, an average value directly proportional to T_a, a sine wave with ω_b angular velocity and amplitude directly proportional to T_b and another with ω_c angular velocity and amplitude directly proportional to T_c. The following steps show what is actually output.

$$\text{The Torquemeter Output} = a_0 + a_1T_{in} + a_2T_{in}^2 \quad (3)$$

Substituting T_{in} in the Output equation, yields

$$\text{Output} = a_0 + a_1[T_a + T_b (\sin \omega_b t) + T_c (\sin \omega_c t)] + a_2[T_a + T_b (\sin \omega_b t) + T_c (\sin \omega_c t)]^2 \quad (4)$$

Simplifying and using trigonometric identities for $(\sin \omega t)^2$ and $(\sin \omega_b t)(\sin \omega_c t)$ yields: 9 terms. An average value (dc) term with error components, the two original sinusoidal terms with the correct amplitude proportionality, those same sinusoids with distorted amplitude proportionality, the second harmonic of each input sinusoid, and sinusoids whose frequencies are the sum and difference of

Polynomial Order →	1	2	3	4	5
a_0	4.757248e-03	1.536968e-03	1.145535e-03	1.054976e-03	8.586305e-04
a_1	2.255475e-06	2.274155e-06	2.279515e-06	2.282194e-06	2.295171e-06
a_2		-1.622418e-14	-2.828503e-14	-3.962629e-14	-1.292879e-13
a_3			6.904549e-21	2.227578e-20	2.341596e-19
a_4				-6.517705e-27	-2.101401e-25
a_5					6.840371e-32
AIC_c	-1.7847e+02	-1.8966e+02	-1.8695e+02	-1.8348e+02	-1.7983e+02

the input signal frequencies. The exact output responses are listed below.

$$a_0 + a_1T_a + a_2T_a^2 + a_2T_b^2/2 + a_2T_c^2/2 \tag{5}$$

(*average torque term.* a_0 can be zeroed out, a_1T_a is the correct average torque output. The other terms are erroneous and cannot be corrected *because they include unknown dynamic signal amplitudes*)

$$+ a_1 T_c (\sin \omega_c t) + 2 a_2 T_a T_c (\sin \omega_c t) \tag{6}$$

(*ω_c sinusoidal terms. 1st term is correct, the 2nd is an error component that cannot be corrected because it includes unknown dynamic signal amplitudes.*)

$$+ a_1 T_b (\sin \omega_b t) + 2 a_2 T_a T_b (\sin \omega_b t) \tag{7}$$

(*ω_b sinusoidal terms. 1st term is correct, the 2nd is an error component.*)

$$- a_2 T_b^2(\cos 2\omega_b t)/2 \tag{8}$$

(*2nd harmonic of ω_b. This is an error term not present in the signal.*)

$$- a_2 T_c^2(\cos 2\omega_c t)/2 \tag{9}$$

(*2nd harmonic of ω_c. This is an error term not present in the signal.*)

$$+ a_2 T_b T_c (\cos (\omega_b - \omega_c) t) - a_2 T_b T_c (\cos (\omega_b + \omega_c) t)$$

(*sum and difference frequencies, both are error components not present in the signal.*)

Thus, the output contains error components added to the correct average value, error components added to both sinusoidal inputs, and sinusoids not in the input signal including second harmonics and sum and difference frequencies.

When more than two signal frequencies are present, additional sum and difference frequencies and harmonics will be output. If only a single signal frequency is present, errors will be generated in the average value, in the amplitude of the signal frequency, and a second harmonic of that frequency will be created.

When the devices Nonlinearity requires higher order terms to accurately represent it then, the number of distortion components increases. For example, when a third order term is required, a third harmonic of each sinusoid is generated –- $\sin^3 (\omega t) = 1/4 (3 \sin (\omega t) - \sin (3 \omega t))$. **To avoid such errors, the Torquemeter must have an inherently linear response. "Linearizing" its response does not eliminate these errors, it compounds them.**

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